

Triangles

Introduction

* **Definitions:**

1. **Triangles:** Any three sided closed figure is called a triangle.

* **Properties:**

- 1) A triangle has three sides, i.e., AB, BC, AC
 - 2) A triangle has three angles, i.e., $\angle A$, $\angle B$, $\angle C$
 - 3) A triangle has three vertices, i.e., A, B, C
 - 4) Three sides and three angles of a triangle are called the six parts i.e., AB, BC, AC, $\angle A$, $\angle B$, $\angle C$
 - 5) The sum of all the three angles of a triangle is 180° i.e. $\angle A + \angle B + \angle C = 180^\circ$.
This property (condition) is called the angle sum property of a triangle.
 - 6) A triangle has three exterior angles corresponding to each interior angle, and each exterior angle is always equal to the sum of the opposite interior angles, i.e.,
i) $\text{ext } \angle A = \angle B + \angle C$ (ii) $\text{ext } \angle B = \angle A + \angle C$ (iii) $\text{ext } \angle C = \angle A + \angle B$.
2. **Similar Figures:** Two figures are said to be similar if their shapes are same.
Ex: 1. Two circles are always similar.
2. Two line segments are always similar.
3. Two acute triangles are always similar
3. **Congruent Figures:** Two figures are said to be congruent if their shapes and sizes are same.
Ex: 1. Two circles are congruent if their radii are same.
2. Two line segments are congruent, if their lengths are same.
4. **Similar Triangles:** If any two angles of a triangle are equal to the corresponding two angles of the other triangle, then the two triangles are said to be similar, and this is called AA similarity.
If two triangles are similar to each other ($\triangle ABC \sim \triangle DEF$), then
1. Their corresponding angles are equal i.e. $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$.
2. The ratios of their corresponding sides are equal i.e. $AB/DE = BC/EF = AC/DF$.
3. The ratio of their areas is equal to the ratios of squares of their corresponding sides i.e. $\text{AI}(\triangle ABC)/\text{AI}(\triangle DEF) = AB^2/DE^2 = BC^2/EF^2 = AC^2/DF^2$ (Ratio Theorem).
5. **Congruent Triangles:** Two triangles are said to be congruent, if they satisfy any one of the condition
SSS (Side, Side, Side): If all the three sides of a triangle are equal to the corresponding three sides of the other triangle, then the two triangles are said to be congruent, and this is called SSS congruency.

SAS (Side, Angle, Side): If two sides and the included angle of a triangle are equal to the corresponding two sides and the included angle of the other triangle, then the two triangles are said to be congruent and this is called SAS congruency.

ASA (Angle, Side, Angle): If any two angles and the included side of a triangle are equal to the corresponding two angles and the included side of the other triangle, then the two triangle are said to be congruent and this is called ASA congruency.

RHS (Right, Hypotenuse, Side): If the hypotenuse and any one side of a right triangle are equal to the corresponding hypotenuse and the side of the other right triangle, then the two triangles are said to be congruent. This is called RHS congruency.

If two triangles are congruent to each other ($\triangle ABC \cong \triangle DEF$), then

1. Their corresponding angles are equal i.e. $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$.
2. The ratios of their corresponding sides are equal i.e. $AB/DE = BC/EF = AC/DF$.
3. The ratio of their areas is equal to the ratios of squares of their corresponding sides i.e. $Ar(\triangle ABC)/Ar(\triangle DEF) = AB^2/DE^2 = BC^2/EF^2 = AC^2/DF^2$ (Ratio Theorem).
4. $Ar(\triangle ABC) = Ar(\triangle DEF)$
5. All congruent triangles are similar.
6. All similar triangles are maybe may not be congruent.
